

Washington University in St. Louis Washington University Open Scholarship

Topics in Quantum Mechanics

Chemistry

Summer 6-9-2016

First Order Time Evolution

Ronald Lovett

Washington University in St. Louis

Follow this and additional works at: https://openscholarship.wustl.edu/chem_papers



Part of the [Chemistry Commons](#)

Recommended Citation

Lovett, Ronald, "First Order Time Evolution" (2016). *Topics in Quantum Mechanics*. 28.
https://openscholarship.wustl.edu/chem_papers/28

This Classroom Handout is brought to you for free and open access by the Chemistry at Washington University Open Scholarship. It has been accepted for inclusion in Topics in Quantum Mechanics by an authorized administrator of Washington University Open Scholarship. For more information, please contact digital@wumail.wustl.edu.

A Simple Mechanical System with First Order Time Evolution

A. The Simple Harmonic Oscillator

The equation of motion for a simple one-dimensional harmonic oscillator,

$$\ddot{x}(t) + \omega^2 x(t) = 0,$$

can be written as two (coupled) first order equations,

$$\begin{aligned}\dot{x}(t) &= v(t), \\ \dot{v}(t) &= -\omega^2 x(t).\end{aligned}$$

B. Replace $\{x(t), v(t)\} \rightarrow z(t)$

If

$$z(t) = x(t) + i \frac{v(t)}{\omega},$$

with $i^2 = -1$,

$$\dot{z}(t) = \dot{x}(t) + i \frac{\dot{v}(t)}{\omega} = v(t) - i \omega x(t) = -i \omega \left[x(t) + i \frac{v(t)}{\omega} \right] = -i \omega z(t)$$

The second order (in time) equation for $x(t)$ has been replaced by a first order (in time) equation for $z(t)$.

C. The simpler equation of motion is easier to solve

Reducing the dynamical equation to

$$\dot{z}(t) = -i \omega z(t) \tag{1}$$

produces a *simpler* mathematical task. If the subscript $_0$ labels initial conditions, the solution of Eq(1) is

$$z(t) = e^{-i\omega t} z_0 \tag{2}$$

D. Go back to $x(t)$ and $v(t)$

In terms of $x(t)$ and $v(t)$, Eq.(2) reads

$$x(t) + i \frac{v(t)}{\omega} = e^{-i\omega t} \left[x_0 + i \frac{v_0}{\omega} \right] = [\cos(\omega t) - i \sin(\omega t)] \left[x_0 + i \frac{v_0}{\omega} \right]$$

Separating the real and imaginary components of this identifies

$$x(t) = x_0 \cos(\omega t) + \frac{v_0}{\omega} \sin(\omega t)$$

$$v(t) = v_0 \cos(\omega t) - \omega x_0 \sin(\omega t)$$

the correct solution to the simple harmonic oscillator dynamical problem.

